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Comparison of Thermodynamic Loss Models Suitable for Gas Turbine Propulsion

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The description of Several figures of merit for estimation of loss in work potential based on the second law of thermodynamics and evaluation of their relative merits for propulsion system analysis and design are the objectives. The loss figures of merit examined are exergy, gas horsepower, and thrust work potential. Definitions and simplified expressions for evaluating each are presented and related via contours on a Temperature-Entropy diagram, and a working comparison is provided by way of a pedagogical example using the J-79 turbojet engine. It is shown that thrust work potential is a special case of gas horsepower, which is in turn a special case of exergy. Based on these results, a general taxonomy is suggested to classify the various work potential figures of merit. The results of this analysis are then used to draw inferences as to what applications each work potential figure of merit is best suited, the general conclusion being that they are complimentary, with each figure of merit being well suited to a particular application. Finally, a general work exclusion principal is suggested as a guide to which of the various loss figures of merit is most appropriate for a given application.

Nomenclature

A	=	cross-sectional area, ft^2
Ae	=	available energy, Btu
C_{FG}	=	nozzle thrust coefficient, equal to ideal/actual gross thrust
c_p	=	constant pressure specific heat, 0.24 Btu/lbm-°R
Ex	=	exergy, Btu
F_{net}	=	net thrust, lbf
g	=	gravitational acceleration, 32.17 ft/s-s
H	=	enthalpy, Btu
I	=	impulse function, lbf
J	=	work equivalent of heat, 778 ft-lb/Btu
M	=	Mach number
$\dot{m}$	=	mass flow rate, lbm/s
P	=	pressure, atm
R	=	gas constant, 0.069 Btu/lbm-°R for air
S	=	entropy, Btu/°R
Sa	=	stream thrust, lbf/lbm
T	=	temperature, °R
u	=	flight velocity, ft/s
V	=	gas velocity, ft/s
W_P	=	thrust work potential, hp or ft-lb/s
$\dot{W}_{out}$	=	power output, hp
γ	=	ratio of specific heats, 1.4
$\Delta P/P$	=	combustor pressure drop, %
ρ	=	gas density, slug/ft^3
η	=	efficiency

Subscripts

amb	=	ambient conditions
C	=	compressor
exp	=	isentropically expanded to ambient pressure
T	=	turbine
th	=	thermal efficiency

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Introduction

THE past several decades have witnessed considerable research and development of methods for estimation of loss in work potential for thermodynamic systems. These methods are based on the combined first and second laws of thermodynamics and have proven to be very powerful analysis tools for estimation of losses for various applications. As pointed out by Bejan,[1] the applications in which work potential methods have the strongest impact are those wherein thermodynamic losses (and minimization thereof) play a pivotal role in determining the design of the system.

One such area where losses are a driving influence is vehicle propulsion system design, particularly aircraft jet propulsion. In fact, from a thermodynamic standpoint, an aircraft in cruising flight produces nothing but aerothermodynamic loss. The crux of the aircraft cruise optimization problem is optimal partitioning of these losses between engine internal losses and drag work (loss) such that the total loss is minimized. It follows, then, that a key step toward the conquest of losses is to have a means of analyzing them in detail so that one can understand where the majority of losses are occurring and focus research and development effort accordingly.

Loss analysis methods have the long-term potential to change the way propulsion systems are analyzed and designed, their chief merit being that they provide powerful insight into the true thermodynamic cost of each loss source. Standard cycle analysis techniques based on the first law of thermodynamics are misleading in this regard because they can only measure the quantity of energy, not the quality (work-producing potential). The second strength of a loss analysis approach is that it puts all losses on an equal (directly comparable) footing. That is to say, all losses are quantified in terms of a loss in work potential, which is a thermodynamic property of the working fluid and not a function of the machine or component itself. Therefore, one can dispense with the concept of component efficiency altogether and simply focus on minimizing absolute loss in all components.

The potential for improved propulsion system designs based on methods using the concept of thermodynamic work potential has prompted considerable interest in discovering means by which to apply the concept to complex systems analysis and design.[2–5] As a result, several models for evaluation of loss in work potential have appeared in the past several decades, each different from the others in subtle ways. Most of the published work in this area focuses on a single model in isolation from the others. As a result, the relationships amongst these various figures of merit (FoM) are ambiguous, and the literature on the subject is disjointed. The purpose of this paper is to clarify this situation by examining the utility of the various loss FoM for gas-turbine propulsion applications and place each in context relative to one another. This paper defines each FoM separately, discusses the historical context, and compares their relative merits for the purposes of jet propulsion. The loss models examined herein are those that have received the majority of attention in open literature, these being exergy, gas horsepower (GHP), and thrust work potential. All three have shown considerable promise as a universal loss metric for jet propulsion applications and warrant investigation.

The first part of this paper will focus primarily on the theoretical background of each FoM whereas the latter portion focuses on application of the three loss models. The latter is done primarily through the use of a pedagogical example modeled on the General Electric J-79 turbojet engine. This example has been intentionally simplified so that the reader can easily verify the results using hand calculations and thereby reinforce understanding of the material presented here. The authors assume some familiarity on the part of the reader with second-law methods of system analysis. Lacking this, the interested reader is referred to the particularly lucid presentation given by Bejan.[6]

Background

There is today a substantial body of work dealing with second-law approaches to measuring loss in gas-turbine engines. One such approach is the exergy concept, which has been applied to the gas-turbine cycle by several authors, notably Clarke and Horlock,[2] who applied it to a simple turbojet example and showed where the most significant exergy losses were occurring. It is the best-known and most formalized method to estimate the magnitude of losses relative to a thermodynamically ideal process.[7,8] It first appeared in the United States due largely to the work of Keenan in the 1940s.[9] A considerable body of literature exists describing the theory and application of exergy analysis, and Refs. 10–13 are standard texts on the subject. More recently, there has been a great deal of interest in applying exergy concepts to combined cycle power generation, of which El-Masri[14] gives an example wherein he is able to identify in detail all sources of exergy loss occurring within a gas-turbine topping cycle. In addition, he shows the impact of cycle changes on total exergy produced and destroyed. Another work potential FoM that has been proposed in the past is GHP (of isentropic expansion), which is used by Nichols[15] as a universal FoM for combustor loss. It is also used extensively as an FoM for gas-generator power output, but has received little attention beyond this limited application. Another FoM was proposed by Curran and Craig[3] based not on energy, but on force (thrust), and is known as the stream thrust concept. This involves calculation of stream thrust potential (also known as specific thrust) at each flow station and optimization of the cycle to deliver the highest stream thrust potential. Later, Riggins[4] extended this concept by introducing the concept of thrust work potential and lost-thrust work potential and showed that optimization of exergy output does not necessarily lead to the best propulsive cycle from a thrust production point of view. Finally, Riggins suggested a modified definition of exergy, which he termed "engine-based exergy," and showed that this modified definition yielded results identical to those obtained through stream thrust methods. The objective of this section is to define, describe, and compare each of these work potential FoM in detail to highlight the differences between each.

Exergy

Exergy is a thermodynamic property describing the maximum theoretical (Carnot) work that can be obtained from a substance in taking it from a given chemical composition, temperature, and pressure to a state of chemical, thermal, and mechanical equilibrium with the environment. The general definition of exergy is given by

$$Ex \equiv H - H_{amb} - T_{amb}(S - S_{amb}) \tag{1}$$

Note that, although energy is a conserved quantity, exergy is not, and it is always destroyed when entropy is produced. Note also that the exergy content of a substance depends on the ambient environment.

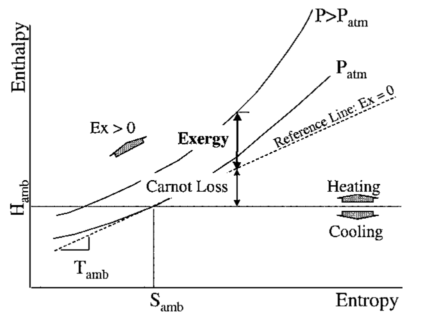

Fig. 1 Definition of exergy plotted on a Mollier diagram.

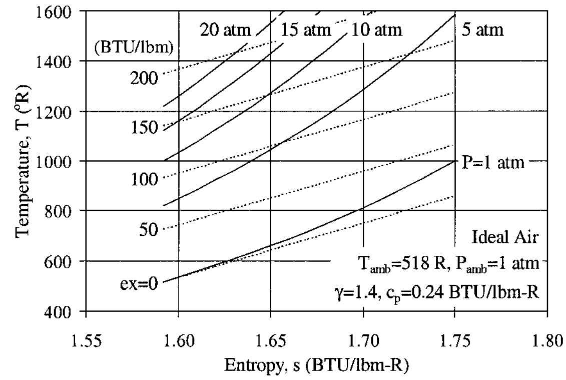

Fig. 2 Contours of constant exergy (- - - -) and isobaric lines (——).

The physical significance of the thermodynamic quantity in Eq. (1) is best described in terms of a Mollier diagram, as shown in Fig. 1. The dashed line with slope equal to the ambient temperature is the zero-exergy reference line that represents the locus of points from which no work can be extracted. All points above this line have the potential to do work via heat transfer from a high-temperature reservoir into the environment. Points below the reference line have potential to do work via heat transfer from the environment into a low-temperature (perhaps cryogenic) reservoir. Also shown are isobaric contours for the reference (ambient) pressure and some arbitrarily higher pressure. Exergy is shown as the difference between the enthalpy delta from the point of interest to the zero exergy reference line. Because the exergy concept relates every state to the Carnot reference of work, change in exergy is a measure of the loss in absolute work potential at every station in an engine. It is a comprehensive measure of loss in work potential that captures the impact of all sources of loss.[16]

Contours of constant exergy for ideal air are plotted on a T–S diagram as shown in Fig. 2. Note that the contours of constant exergy are straight lines with the zero exergy contour passing through the ambient state. Also, it is clear from Fig. 2 that the zero exergy contour is tangent to the ambient pressure contour at ambient conditions. For the case of calorically perfect air where chemical potential, kinetic energy, and potential energy are negligible, it is a simple matter to obtain an equation for mass-specific exergy as a function of ambient conditions and gas conditions at a given engine station by noting that

$$h - h_{amb} = c_p(T - T_{amb}) \tag{2}$$

and using the integrated form of the second TdS relation

$$s - s_{amb} = c_p \ln(T/T_{amb}) - R \ln(P/P_{amb}) \tag{3}$$

(Note that lowercase thermodynamic variables denote mass-specific quantities). Substitution of Eq. 2 and Eq. 3 into Eq. 1 yields

$$ex = c_p(T - T_{amb}) - c_p T_{amb} \ln(T/T_{amb}) + R T_{amb} \ln(P/P_{amb}) \tag{4}$$

Application of this equation to every engine station yields the exergy at each station. The losses associated with each component can then be calculated based on the idea that the difference between the exergy fluxes into and out of a component must be equal to the sum of the power output and the exergy loss rate

$$\dot{E}x_{\mathrm{in}} - \dot{E}x_{\mathrm{out}} = \dot{W}_{\mathrm{out}} + \dot{E}x_{\mathrm{loss}} \tag{5}$$

Exergy is the most general of the work-potential FoM investigated here and gives an estimate of the absolute work potential that could be obtained by any heat engine operating under specified conditions. The work potential estimates obtained using exergy assume that the working substance is taken to thermal and mechanical equilibrium with the environment, the first of which cannot be enforced using the simple Brayton cycle. As a result, even the perfect (no component loss) Brayton cycle will have exergy losses due to nonequilibrium combustion and exhaust heat loss. Moreover, when the objective is to produce jet thrust, a portion of the work done on the working fluid must appear as exhaust residual kinetic energy as viewed in the Earth-fixed reference frame. Thus, there is a portion of the exergy content of the fuel that is inherently unavailable to the Brayton cycle and appears as a loss. In general, these inherent losses are far larger than exergy losses due to component inefficiencies for gas-turbine engines. Consequently, optimization of a thrust-producing device to produce maximum exergy output may yield a less-than-optimal result if the objective is to produce thrust for propulsion, as observed by Riggins.[4]

However, for some applications such as combined-cycle power generation, it appears that there is justification for optimizing exergy output to obtain maximum power output. The reason for this is that the steam-bottoming cycle is able to extract the exhaust exergy of the gas-turbine topping cycle, and therefore, all of the exhaust exergy becomes inherently available. One would, thus, expect that optimization of the topping cycle for maximum exergy output (in the form of shaft power and exhaust exergy) will naturally lead to more efficient combined-cycle plants.

GHP

GHP is defined as the work that would be obtained by isentropically expanding a gas at a specified temperature and pressure to a prescribed reference pressure (usually taken to be local atmospheric). Thus, the temperature at the imaginary expanded condition is fallout from the isentropic expansion process. Expressed mathematically

$$\mathrm{GHP} \equiv H(T_i, P_i) - H(P = P_{\mathrm{ref}}, S = S_i) \tag{6}$$

where i is the thermodynamic state of the gas at point i. Note that GHP is a function of ambient pressure, independent of ambient temperature, unlike exergy. Gas horsepower is commonly used to measure the theoretical power output of core engines and gas generators and has been referred to variously as available energy[15] or barergy. A simple expression for GHP of a calorically perfect gas is easily derived by noting that

$$\mathrm{GHP} = c_p\{T_i - T(P_{\mathrm{amb}}, S = S_i)\} \tag{7}$$

where the temperature after isentropic expansion to reference (ambient) pressure is

$$T(P_{\mathrm{amb}}, S = S_i) = T(P_{\mathrm{amb}}/P)^{(\gamma-1)/\gamma} \tag{8}$$

combining Eq. (7) and Eq. (8) yields

$$\mathrm{GHP} = c_p T\left[1 - (P_{\mathrm{amb}}/P)^{(\gamma-1)/\gamma}\right] \tag{9}$$

Application of Eq. (9) to the results of a standard first law cycle analysis yields the GHP at each station in the engine. These results can then be used to calculate the loss in GHP in each component of the engine based on a conservation of GHP principle similar to that used for calculation of exergy losses:

$$\dot{\mathrm{GHP}}_{\mathrm{in}} - \dot{\mathrm{GHP}}_{\mathrm{out}} = \dot{\mathrm{W}}_{\mathrm{out}} + \dot{\mathrm{GHP}}_{\mathrm{loss}} \tag{10}$$

One can obtain a physical feel for the meaning of Eq. (6) by comparison to the definition of exergy, as expressed in terms of a

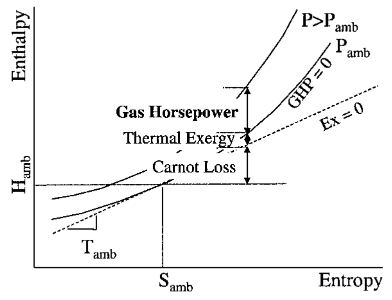

Fig. 3 GHP plotted on a Mollier diagram.

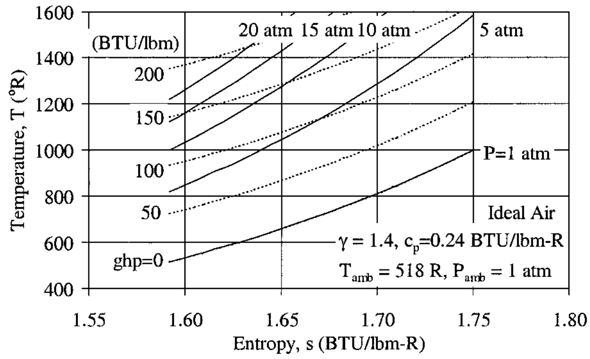

Fig. 4 Contours of constant GHP (- - - -) and isobaric lines (——).

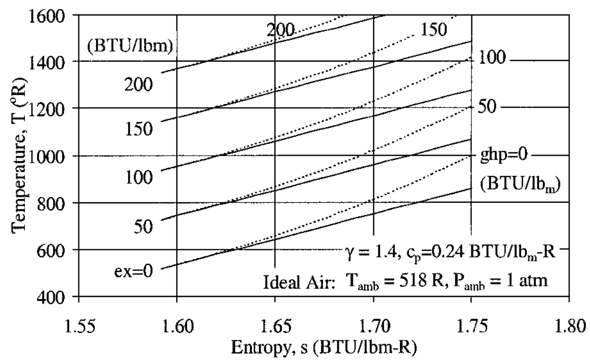

Fig. 5 Contours of constant exergy (——) and constant GHP (- - - -).

Mollier diagram, as shown in Fig. 3. Note that whereas the contour of zero exergy is a line passing through the ambient state, the contour of zero GHP is the isobaric line corresponding to ambient pressure. The space between the zero exergy line and the zero GHP contour is labeled as thermal exergy and constitutes the difference between GHP and exergy. Clearly, GHP and exergy are identical at the ambient entropy state, but diverge as entropy increases. This idea is further explained by plotting lines of constant GHP on a T–S diagram, as shown in Fig. 4. Note that as temperature and pressure increase above the ambient value, the contours of constant GHP diverge from the isobaric contours. Also, it is clear from Eq. (9) that the GHP is directly proportional to the gas temperature for a given pressure ratio (for the calorically perfect gas model only).

It was pointed out earlier that GHP and exergy are thermodynamically identical quantities at the reference (ambient) entropy and diverge with increasing entropy. This is also reflected in the T–S diagram, shown in Fig. 5, which shows lines of constant exergy and GHP superimposed on the same plot for ideal air. The fundamental difference between these two quantities is that GHP requires only mechanical (pressure) equilibrium with the environment, whereas exergy requires both mechanical and thermal equilibrium. Thus, GHP is a special case of exergy for which work extraction through mechanical equilibrium is assumed while thermal equilibrium is not enforced.

Table 1 Taxonomy of prominent work potential FoM

Property	Thermal equilibrium	Mechanical equilibrium	Mixture equilibrium	Chemical equilibrium	Nuclear equilibrium
Exergy	×	×	×	×	?
GHP		×			
Thrust work potential		×			
Energy source	Internal energy contained in ensembles of particles		Molecular diffusion	Chemical bonds	Nuclear bonds
General scale of effect	Macroscopic ensembles of particles			Molecular scale	Atomic scale

GHP losses in a given component will always be larger than their corresponding exergy losses. This is because the loss in work will appear as an increase in heat (by the first law) and temperature of the exhaust gas. A portion of this exhaust heat is recoverable and can be used to produce work in a bottoming cycle and is thus not seen as an irretrievable loss using exergy, but is an unrecoverable loss in GHP.

The primary difference between GHP and exergy for analysis of gas-turbine (Brayton) cycles is that exhaust heat does not appear as a GHP loss, but is instead transparent. Thus, a perfect Brayton cycle will appear to have no losses in GHP. Therefore, the distribution of losses is quite different between the exergy and GHP methods. In particular, the role of component losses and useful work production is considerably magnified because nonequilibrium combustion and exhaust heat losses no longer appear in the loss stackup. However, the GHP FoM still shows exhaust residual kinetic energy as a large loss when the gas-turbine is used to produce jet thrust.

Thrust Work Potential

Thrust work potential is defined as the thrust work that would be obtained in expanding a flow at a given temperature and pressure to ambient pressure such that the thrust work obtained is equal to the thrust produced multiplied by the flight velocity of the aircraft.[17] This can be normalized by airflow rate to give specific thrust work potential at each station

$$W_p \equiv Sa(u)/J \tag{11}$$

where Sa is stream thrust, which is defined as the impulse per unit mass flow rate.[18] It is, therefore, related to impulse function[19] by

$$Sa \equiv I/\dot{m} \tag{12}$$

It is generally preferable to evaluate the stream thrust in terms of temperatures and pressures at each flow station in the engine. This is done by calculating the velocity that would be obtained by an imaginary isentropic expansion from the conditions of interest to atmospheric pressure, where the expanded fluid velocity is related to the stream thrust via

$$Sa = V_{\text{expanded}}/g \tag{13}$$

This expanded velocity can be evaluated based on the GHP at each flow station

$$Sa = \sqrt{2(\text{GHP})J/g} \tag{14}$$

For the purposes of airbreathing propulsion, thrust work potential is inherently anchored in the Earth-fixed observer's frame of reference because it is based on the velocity of the vehicle relative to the Earth. Note that the thrust work potential is always less than the GHP of the gas stream because some of the GHP must necessarily emerge as residual kinetic energy of the exhaust gases (as viewed by the stationary observer). Thrust work potential is, therefore, linked to the GHP through propulsive efficiency, which is in turn a function of exhaust velocity and flight velocity. In this regard, thrust work potential can be viewed as a special case of GHP that measures only work produced with respect to a particular reference frame. By extension then, thrust work potential is a special case of exergy.

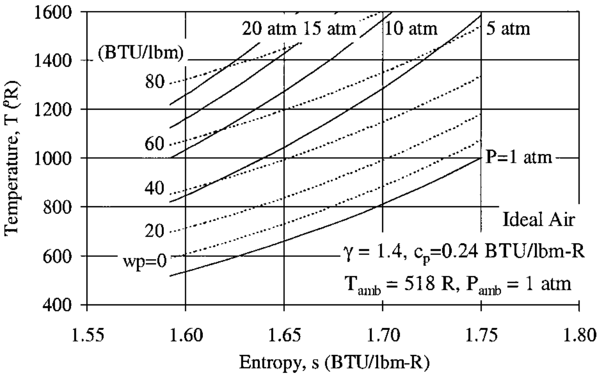

Fig. 6 T–S diagram with lines of constant thrust work potential (- - - -) at M = 0.9, 20,000-ft altitude and isobaric lines (——).

Lines of constant thrust work potential can be plotted on a T–S diagram, as shown in Fig. 6. Note that the contours are shaped the same as GHP contours, but with two differences. Their spacing is not constant, and the zero thrust work potential line does not coincide with the zero-GHP (atmospheric pressure) line. In fact, the spacing of thrust work potential contours is proportional to the square of the temperature because of its relationship to stream thrust. The displacement of the zero point physically corresponds to the thrust work required to offset the ram work of inlet compression. Thus, the zero thrust work potential line will move further upward as flight velocity increases. It follows, then, that thrust work potential for a rocket is the same as shown in Fig. 6, except that the zero thrust potential line is equal to the zero GHP line for all flight velocities. Finally, note that there is far less thrust work potential available than GHP for a given flow temperature and pressure, especially at high temperature and pressure. This is due to increasing exhaust residual kinetic energy (lower propulsive efficiency), which is characteristic of the high specific thrust produced by high enthalpy flows.

Thrust work potential has the disadvantage that it is not a meaningful FoM for comparison of engines at static operation. However, because losses are expressed in terms of power, it can be directly compared against exergy and GHP methods. In addition, thrust work potential does not count exhaust residual kinetic energy as being available for propulsive purposes. Therefore, it is not accounted as a loss, unlike exergy and GHP.

Taxonomy of Work Potential FoM

Based on the preceding discussion, it appears profitable to construct a rough taxonomy of the various loss FoM and to classify them relative to one another, as shown in Table 1. An × is placed in the appropriate matrix cells to indicate which sources of work potential are accounted for by each method. Table 1 is by no means exhaustive and is only intended to cover those concepts that appear to have the most use as a propulsive work FoM. Because this is a relatively new and maturing field, the definitions given in Table 1 are the authors' interpretation of each FoM, and other authors have offered alternative definitions to those given here. None of the work potential FoM discussed here have been extended to account for the potential available in nuclear and subnuclear bonds. The authors are unaware of any FoM that captures this aspect of work potential, and this is apparently an area for future theoretical exploration in the physics, thermodynamics, and power-generation communities.

Based on the discussion thus far, the relationship between exergy, GHP, and thrust work potential becomes relatively clear. Thrust work potential is nothing more than a measure of the propulsion system's ability to project thrust work into another arbitrary (usually Earth-fixed) reference frame. It is a special case of GHP to which it is related through propulsive efficiency. GHP is merely a special case of exergy wherein only mechanical equilibrium with the environment is enforced.

Simplified J-79 Analytical Model

The objective of this section is to give a practical demonstration and comparison of loss models for a simplified turbojet engine cycle model based on the J-79.[20–22] The assumptions used in this analysis are enumerated in Table 2, and a schematic of the assumed engine configuration and station nomenclature is shown in Fig. 7. Note that station 4a is an imaginary station immediately before heat addition but after the combustor pressure drop, inserted to artificially separate the combustor heat addition and pressure drop processes. Two operating conditions are considered here for analysis: sea-level static military power and Mach 0.9, 20,000-ft altitude military power. These two operating conditions were chosen because they are nearly identical in that the engine front face experiences the same temperature and nearly the same static pressure for both cases. Thus, if Reynolds number and nozzle pressure ratio effects are ignored, the component efficiencies are the same for both conditions.

The model used here considers only four sources of component loss: compressor efficiency, turbine efficiency, nozzle thrust coefficient, and combustor pressure drop. The presence of secondary cooling flow circuits, mechanical losses, leakage, customer power takeoff, customer bleed, etc., is ignored as these would merely distract from the objective. It is relatively straightforward to incorporate these effects into the cycle model if a detailed loss analysis is desired.

Based on these assumptions, a standard cycle analysis[23] using conservation of mass, momentum, and energy yields the J-79 performance estimates given in Table 3, and the internal temperatures

Table 2 Assumptions used for analysis of simplified J-79 cycle model

Engine cycle	Overall pressure ratio = 13.5
	Turbine inlet temperature = 1830 F (2290°R)
	Airflow = 170 lbm/s (132.3 lbm/s at 20 K, $M = 0.9$)
Efficiencies	Compressor efficiency = 0.85
	Turbine efficiency = 0.90
	Combustor pressure drop = 5%
	Nozzle thrust coefficient = 0.985
	All other components are perfect
	No internal cooling flow circuits
Other	No installation effects, AIA ram recovery
	Dry (unaugmented) operation only
	Calorically perfect gas (γ = const = 1.4)
	Constant gas composition (no vitiation)
	Fuel mass is negligible relative to air mass
	Gas kinetic energy inside engine is negligible
	Reference conditions are local ambient
	Nozzle exit pressure is local ambient
	$c_p = 0.24$ Btu/lbm, fuel heat = 18,400 Btu/lbm

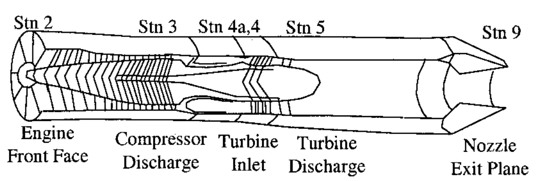

Fig. 7 Simplified J-79 cycle model configuration and station designations per Society of Automotive Engineers ARP755B.

Table 3 J-79 performance estimates based on standard cycle analysis

FoM	Sea-level static	$M = 0.9$, 20,000 ft
Air flow rate, lbm/s	170.0	132.3
Compressor power required, Btu/lbm	161.4	162.0
Thermal energy input, Btu/lbm	263.9	262.8
Fuel/air ratio	0.0148	0.0147
Fuel flow rate, lbm/h	9,047	7,010
SFC , lbm/lbf-h	0.740	0.974
Thermal efficiency, %	40.5	54.8
Gross thrust (dry), lbf	12,227	11,034
Ram drag, lbf	0	3,837
Net thrust, lbf	12,227	7,198
Gas power output[a], hp	25,717	26,923
Gross thrust power, hp	0	18,724
Net thrust power, hp	0	12,214
Propulsive efficiency, %	0	45.4
Overall efficiency, %	0	24.8

[a]Assuming turbine expansion to ambient pressure in vehicle-fixed frame.

and pressures listed in Table 4. These results show that the thermal efficiency of the J-79 engine estimated for sea-level static conditions is 40.5%. This efficiency appears to be quite low, leading one to believe that the configuration is a poor design from an energy utilization standpoint and could be improved on tremendously. However, this perception is misleading because it only describes the distribution of energy, not the distribution of work potential. In fact, as will be shown using the various second-law methods, the actual power output for this engine is on the order of 75–80% of the thermodynamic ideal available using the Brayton cycle with a 13.5 pressure ratio. Also, note that the specific fuel consumption (SFC) of this example engine is optimistic relative to the actual machine due to the simplifying assumptions used in the analysis.

The thermal efficiency of the engine operating at Mach 0.9 and 20,000 ft is somewhat higher due to the lower ambient temperature as well as the higher overall cycle pressure ratio imparted via ram compression in the inlet. Note that the engine produces more power (GHP) at Mach 0.9 than sea-level static. Thrust horsepower is measured relative to an Earth-fixed reference frame, as is net thrust horsepower for all cases discussed herein.

Perturbation Estimates for Component Loss

The most common technique used today for gauging the relative importance of losses in gas-turbine engines is a general class of methods referred to here as perturbation methods. These methods involve the analysis of a family of cycles, each derived from a common baseline cycle. The objective is to perturb the baseline, one parameter at a time, and analyze the impact that this has on performance. The change in perturbed performance relative to the baseline is then used to deduce the total loss contributed by an individual loss mechanism.

There are two broad categories of perturbation method. The first involves infinitesimal perturbations from a baseline to obtain what are commonly referred to as sensitivity derivatives. The other method involves large (usually nonlinear) perturbations such that component interactions cannot be considered insignificant. Small-perturbation methods cannot be used to calculate directly the absolute magnitude of losses, though it is possible to gauge the relative importance of loss mechanisms through the use of sensitivities. Large-perturbation methods, on the other hand, can be used to approximate component loss directly. This is done by simply resetting one component efficiency at a time to 1.0 and rebalancing the cycle. The difference between the baseline and the perturbed machine performance is then taken to be the impact due to losses in the component that was perturbed.

The results of a large-perturbation analysis of the J-79 are shown in Table 5. The assumption used here is that the compressor discharge temperature should be held constant while the cycle pressure ratio is allowed to vary. The results from this analysis indicate that compressor losses are dominant, followed by turbine, nozzle, and

Table 4 Engine internal temperature and pressure for each flow station

		Sea-level static		$M = 0.9$, 20,000 ft	
Description	Station	Temperature, °R	Pressure, atm	Temperature, °R	Pressure, atm
Freestream	0	519	1.00	447	0.46
Inlet flange	2	519	1.00	519	0.78
Compressor discharge	3	1191	13.50	1195	10.50
Turbine inlet	4a	1191	12.83	1195	10.00
Turbine inlet (with $\Delta P/P$)	4	2290	12.83	2290	10.00
Turbine discharge	5	1617	3.23	1615	2.50
Nozzle exit plane	9	1172	1.00	1015	0.46

Table 5 Approximate absolute impact of engine component losses based on perturbed component efficiencies at M = 0.9, 20,000-ft altitude

Component loss	Change in thrust, lbf	Change in SFC, lb/lb-h	Change in η_{th}, %
Compressor efficiency 100%	+750.7	−0.090	+7.57
Combustor $\Delta P/P$: 0%	+127.3	−0.017	+1.17
Turbine efficiency 100%	+417.2	−0.052	+4.09
Nozzle C_{fg}: 100%	+170.0	−0.022	+1.58

Table 6 Exergy at each engine station relative to the vehicle-fixed reference frame

	Sea-level static		$M = 0.9$,20,000 ft	
Station	Ex, Btu/lbm	ΔEx, Btu/lbm	Ex, Btu/lbm	ΔEx, Btu/lbm
Freestream	0	——	17.4	——
2	0	0.0	17.4	0.0
3	150.4	+150.4	169.9	+152.5
4a	148.6	−1.8	168.3	−1.6
4	331.1	+182.6	361.3	+193.0
5	163.8	−167.3	194.2	−167.1
9	162.4	−1.5	192.2	−2.0
Exhaust	0	−162.4	0	−126.9

combustor losses. The caveat to using this method is that it changes the cycle of the machine. For instance, if compressor efficiency is set to 1.0, the cycle will rebalance either at a higher pressure ratio for the same compressor power input or will have reduced shaft power input to the compressor for the same pressure ratio. Either way, the cycle is fundamentally changed and so the loss estimate obtained via this method is part due to component loss and part due to a change in the cycle (this applies to sensitivity methods also). Consequently, the best methods presently available to estimate component loss can only yield approximations in which the true component loss is inevitably confounded with interactions between various engine components due to the rebalancing process.

Exergy

The results of the exergy analysis for the simplified J-79 are shown in Tables 6 and 7. Note that the largest exergy losses are nonequilibrium (irreversible) combustion and exhaust heat rejection, both of which are due to the nature of the Brayton cycle itself rather than an imperfection in the machine. The only way these losses can be reduced is either through a change in the cycle parameters (in this case, overall pressure ratio and turbine inlet temperature) or a change in the basic configuration of the machine (such as the addition of a regenerator). This result clearly shows that there is considerable room for improvement relative to the theoretical ideal. Among the component-specific losses, the compressor is the most significant contributor, followed by the turbine. This result is because 1) the compressor efficiency is lower and 2) inefficiencies at low temperature tend to be more significant in an exergy sense than those at high temperature.

The overall exergy input for the SLS cycle is approximated as 263.9 Btu/lbm and the power output (in the form of exhaust kinetic energy) is 106.9 Btu/lbm, yielding a thermal efficiency of 40.5%. One could argue that because irreversible combustion and exhaust heat losses are inevitable for the Brayton cycle, they should not be chargeable against the machine itself. Viewed in this light, the effective exergy input is the total exergy input less the irreversible combustion and exhaust heat losses (127.1 Btu/lbm), yielding a Brayton-corrected thermal efficiency of roughly 84%. In effect, this means that 84% of the exergy that is theoretically accessible using the nominal J-79 cycle was converted into exhaust kinetic energy, with the remainder being dissipated as exhaust heat (this neglects that component inefficiency contributes to exhaust heat also).

Note that if the J-79 example were used as a topping cycle in a combined cycle plant, it would then be possible to convert the exhaust heat exergy from the J-79 powerplant into useful work produced by the bottoming cycle. In this case, it would clearly be appropriate to count exhaust exergy as a loss because it is theoretically available to the machine, although the exergy loss due to irreversible combustion would still be nonchargeable. In this case, the combined cycle corrected efficiency would be 106.9/(263.9-81.3) or 58.5%.

Finally, if this same fuel were combusted in a fuel cell instead of a gas-turbine engine, it would be possible to combust the fuel at near-equilibrium conditions and thereby avoid the losses due to irreversible combustion in a gas-turbine engine. The total exergy theoretically accessible by the fuel-cell powerplant is roughly equal to the Gibbs free energy of the reaction, which typically approaches 90% or more of the total heating value. In this scenario, the first-law cycle efficiency of 40.5% begins to have real meaning because it is theoretically possible to approach 100% efficiency with such a machine. This line of reasoning suggests that the appropriate choice of thermal efficiency depends in some measure on the nature of the machines being used and the intent of the analyst.

GHP

The GHP results for the J-79 example shown in Tables 8 and 9 indicate that the turbine incurs the greatest loss from a GHP viewpoint, followed by the compressor, combustor pressure loss, and finally the nozzle. Not surprisingly, the component losses estimated using the GHP FoM are larger than their exergy counterparts. This is because losses contribute to increased exhaust gas temperature and, therefore, the exhaust exergy of the flow. Thus, a portion of the loss is recoverable in the exhaust heat and is not counted as a loss using the exergy method. All results for GHP are assumed to be in the vehicle-fixed reference frame, that is, the reference frame of the observer moving with the propulsion system.

Note that irreversible combustion and exhaust heat losses do not appear in this analysis (they are listed in Table 9 as nonchargeable to denote that they are not charged against the machine efficiency). If all component efficiencies in this engine were perfect, the total GHP loss would be zero, and the total power output would be equal to the ideal cycle. Based on this observation, it is reasonable to view GHP as a Brayton FoM because GHP is completely accessible within the confines of the simple Brayton cycle.

The implication of the preceding statement is that GHP is a good FoM for engines designed to produce shaft horsepower such

Table 7 Sources of exergy loss

	Sea-level static			$M = 0.9$, 20,000 ft		
Component	Exergy loss, Btu/lbm	Btu/lbm	Loss/Ex_{in}, %	Exergy loss, Btu/lbm	Btu/lbm	Loss/Ex_{in}, %
Compressor	161.4 − 150.4	11.0	4.2	162.0 − (169.9 − 17.4)	9.5	3.6
Combustor $\Delta P/P$	150.4 − 148.6	1.8	0.7	169.9 − 168.3	1.6	0.6
Irreversible combustor	263.9 − 182.6	81.3	30.8	262.8 − 193.0	69.8	26.6
Turbine	(331.1 − 163.8) − 161.4	5.9	2.2	361.3 − 194.2 − 162.0	5.1	1.9
Nozzle	163.8 − 162.4	1.4	0.5	194.2 − 192.2	2.0	0.8
Exhaust heat	162.4 − 106.9	55.5	21.0	192.2 − 143.9	48.3	18.4
Exhaust residual KE[a]	106.9 − 0	106.9	40.5	143.9 − 17.4 − 65.3	61.2	23.3
Thrust work[a]	12,212 lbf × 0 ft/s	0.0	0.0	7,198 lbf × 933 ft/s	65.3	24.8
Work out/Exergy in	0/263.9	0%		65.3/262.8	24.8%	

[a]As viewed in the Earth-fixed reference frame.

Table 8 GHP at each engine internal station

	Sea-level static		$M = 0.9$, 20,000 ft	
Station	GHP, Btu/lbm	ΔGHP, Btu/lbm	GHP, Btu/lbm	ΔGHP, Btu/lbm
Freestream	0.0	——	0.0	——
2	0.0	——	17.4	+17.4
3	149.9	+149.9	169.4	+152.0
4a	147.9	−2.0	167.7	−1.7
4	284.5	+136.6	321.4	+153.7
5	110.2	−174.2	148.3	−173.1
9	106.9	−3.3	143.9	−4.4
Exhaust	0	−106.9	0.0	−143.9

as turboshaft and simple-cycle gas-turbine power generation units. However, for propulsive devices designed to produce thrust by action on a body of fluid, it is not possible to use all of the GHP produced by the thermal cycle for the production of thrust work (except in the limit of infinitesimal delta velocity acting on an infinite mass flow rate). Thus, aircraft engines not only have an inherent thermal unavailability due to the cycle, they also have an inherent propulsive unavailability as well.[24] Therefore, it seems intuitive that a good FoM for jet propulsion must necessarily involve thrust itself.

Thrust Work Potential

Thrust work potential is a measure of the thrust work obtainable in a simple isentropic expansion, as explained in Ref. 4, and is related to GHP through propulsive efficiency. However, direct estimation of aerothermodynamic losses based on thrust work potential is somewhat ambiguous because there is no conservation of thrust work principle as there is for exergy and GHP. The method suggested by Riggins[4] for estimating loss in stream thrust is to remove progressively the losses from downstream to upstream, rebalancing the cycle after each loss is removed to find change in engine net stream thrust. This method (called the lost thrust work potential method) assumes that the change in thrust work after each step is due to the loss just removed. The logic of this approach can be explained as follows. Losses produced by most components are a function of the upstream flow conditions feeding the component. Therefore, if the objective is to evaluate the loss contributed by a particular component, that component must be evaluated relative to its ideal at the actual inlet conditions. This dictates that losses must be removed from back to front, as the opposite direction would cause changes in the cycle to propagate downstream and interfere with loss estimates for downstream components.

A drawback to using this method for gas-turbine engines is that the loss estimate obtained depends on the assumptions used in rebalancing the cycle. For instance, assume that the cycle pressure ratio and turbine inlet temperature are held constant as losses are removed. When the cycle is rebalanced after removing the compressor losses, the combustor heat input will change because the compressor discharge temperature decreased. Thus, the change in specific thrust is due to not only the compressor loss, but also due to the change in combustor heat addition. Alternatively, if compressor discharge and turbine inlet temperatures are assumed to define the nominal cycle, the compressor loss estimate is due to changes in compressor loss and overall cycle pressure ratio. Therefore, it is important to choose these assumptions such that they are physically meaningful to the problem at hand.

The latter assumption was chosen as being more physically meaningful for this problem because compressor discharge temperature is a technology limit on cycle pressure ratio. The thrust work potential at each engine station is shown in Table 10. The results for lost thrust work potential are as shown in Table 11 and indicate that the compressor causes the greatest loss in thrust work potential, followed by the turbine. The combustor pressure drop and nozzle loss are a distant third and fourth. Note that nonequilibrium combustion, exhaust heat, and exhaust residual kinetic energy losses are transparent as viewed from a thrust work potential perspective. Inspection of these loss estimates also reveals that they are very similar in relative magnitude to the approximate first-law estimates given in Table 5. Note that sea-level static conditions reduce to the trivial case for this FoM and, therefore, are not shown. If one were to define thrust work effectiveness as the ratio of the actual thrust work potential divided by that of the perfect (no component losses) machine, the thrust work effectiveness of the J-79 example is 83.8%.

Comparison of Results

Based on the discussion, one can now make a direct comparison of the various FoM and use this to draw inferences as to the relative merits of each for use in gas-turbine loss estimation. Comparative results for the loss stackup of J-79 work potential are shown in Fig. 8. Note that irreversible combustion and exhaust heat losses only appear in the exergy analysis and are nonchargeable when using the GHP and thrust work potential methods. Exhaust residual kinetic energy is accounted for as a loss for the exergy and gas horsepower methods, but is nonchargeable when using the stream thrust and thrust work potential methods. Based on this comparison, it is clear that the thrust work potential method yields a loss stackup that closely matches the loss stackup obtained using the large-perturbation method. The differences between the perturbation results and thrust work potential results arise because the former is an approximation whereas the latter is an exact analytical result.

It is clear from this comparison that there is much more work potential available in the fuel from an exergy or GHP viewpoint than from a thrust work potential or cycle perturbation point of view. For instance, the exergy input of the fuel is approximately 263 Btu/lbm airflow, whereas the analogous figure for thrust work potential is only 78.0 Btu/lbm air. This is because irreversible combustion, exhaust heat, and residual kinetic energy losses are roughly an order of magnitude greater than individual component losses. Yet, this is not widely recognized, primarily because these losses do not appear when using standard cycle analysis methods to estimate loss. This inherent recognition and comprehensive accounting of all sources of loss is a strong argument for application of exergy methods to propulsion systems analysis.

Note that if the nominal specific work is divided by fuel/air ratio, the result is work potential per pound of fuel. It is a simple matter to

Table 9 Loss in GHP

	Sea-level static			$M = 0.9$, 20,000 ft		
Component	GHP loss, Btu/lbm	Btu/lbm	Loss, %	GHP loss, Btu/lbm	Btu/lbm	Loss, %
Compressor	161.4 − 149.9	11.5	8.4	162.0 − (169.4 − 17.4)	10.0	6.5
Combustor ΔP/P	149.9 − 147.9	3.9	1.5	169.4 − 167.7	1.7	1.1
Nonequilibrium combustor	Nonchargeable	——	——	Nonchargeable	——	——
Turbine	174.2 − 161.4	12.8	9.4	321.4 − 148.3 − 162.0	11.1	7.2
Nozzle	110.3 − 106.9	3.4	2.5	148.3 − 143.9	4.4	2.9
Exhaust heat	Non-chargeable	——	——	Nonchargeable	——	——
Residual KE[a]	106.9 − 0	106.9	78.3	143.9 − 65.3 − 17.4	61.2	39.8
Thrust Work[a]	12,227 lbf * 0ft/s	0.0	0.0	7,198 lbf * 933 ft/s	65.3	42.5
Work out/*ae* in	0/136.6	0.0		65.3/153.7	0.425	

[a] As viewed in the Earth-fixed reference frame.

Table 10 Specific gross thrust work potential for $M = 0.9$, 20,000-ft flight conditions

Station	W_p, Btu/lbm	ΔW_p, Btu/lbm
Ambient	0.0	——
2	34.8	+34.8
3	108.4	+73.6
4a	107.9	−0.5
4	149.6	+41.7
5	101.7	−47.9
9	100.1	−1.6
Net W_p	100.1 − 34.8	65.3

Table 11 Loss in specific thrust work potential for $M = 0.9$, 20,000-ft flight conditions

Component	W_p loss, Btu/lbm	Btu/lbm
Compressor	78.0 − 71.8	6.2 (7.9%)
Combustor $\Delta P/P$	71.8 − 70.6	1.2 (1.4%)
Nonequilibrium combustor	Nonchargeable	——
Turbine	70.6 − 66.9	3.7 (4.8%)
Nozzle	66.9 − 65.4	1.5 (2.0%)
Exhaust heat	Nonchargeable	——
Residual KE	Nonchargeable	——
Useful *wp*	——	65.4
wp out/ideal	65.4/(65.4 + W_p lost)	83.8%

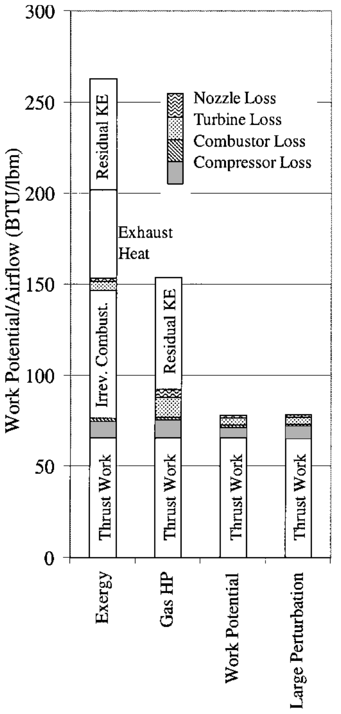

Fig. 8 Absolute specific work output (and loss) per unit airflow for the J-79 turbojet engine at Mach 0.9 and 20,000 ft.

define fuel flow chargeability based on the loss breakdowns shown in Fig. 8. For instance, Table 7 suggests that 26.6% of total fuel flow rate at Mach 0.9 at 20,000-ft altitude is chargeable to irreversible combustion losses, 18.4% is chargeable to exhaust heat, etc. As a result, one can use these loss models to analytically calculate total fuel cost due to compressor losses integrated through a nominal aircraft mission.[25] Likewise, total fuel cost due to turbine losses, inlet losses, aerodynamic drag, etc., can be directly estimated,[5] a capability that does not currently exist using standard analysis techniques.

In summary, these results indicate that both exergy and GHP are physically intuitive and possess a conservation property that allows direct estimation of losses via Eqs. (5) and (10). However, neither produces results that are reflective of the true costs due to component losses in jet propulsion applications because they account for work potential sources that are inherently unavailable to jet propulsive machines.[4] Thrust work potential, on the other hand, accounts for jet propulsive losses in a physically realistic way such that losses due to non-equilibrium combustion, exhaust heat, and exhaust residual kinetic energy (KE) are transparent (non-chargeable) for the thrust work potential model. Exclusion of these losses can be likened to the idea of sunk costs in economics, wherein a cost that is irretrievably spent is ignored when making a decision as to allocation of scarce resources. Likewise, if the propulsion system cycle is specified, then one could view irreversible combustion, exhaust heat, and residual KE as sunk costs incurred in producing jet thrust. However, thrust work potential does not possess a conservation property analogous to Eqs. (5) and (10) for direct estimation of loss.

Conclusions

The loss estimation methods investigated here present a robust suite of tools that can be easily applied to the analysis of propulsion systems, each well suited to a particular application. Their application to the J-79 example revealed that the largest losses in work potential are due to irreversible combustion, exhaust heat, and exhaust residual KE. From a GHP standpoint, the largest loss is due to exhaust residual KE, with turbine and compressor losses being a distant second and third, respectively. As viewed using stream thrust or thrust work potential, the largest losses are due to the compressor, followed by the turbine. The results obtained for this last method are strikingly similar to those obtained using standard perturbation methods and show that roughly 85% of the total work theoretically available in a Brayton cycle having the J-79's pressure ratio and turbine inlet temperature is actually realized in the J-79 engine. Based on these results, one could argue that the most appropriate FoM for jet propulsion purposes is thrust work potential when the objective is component optimization to achieve maximum thrust output for a given thermodynamic cycle. If the engine cycle is taken to be rubberized, then the exergy results will likely be of more interest than thrust work potential results.

The upshot of this comparison is that exergy can be thought of as a Carnot FoM in that a Carnot cycle will appear to have no losses when analyzed using exergy methods, whereas any departure from a Carnot cycle will appear as a loss in exergy. It is the most comprehensive and consistent loss FoM examined in that it can be shown to capture the effect of all losses relevant to contemporary propulsive cycles, including nonequilibrium combustion, exhaust heat, and exhaust residual KE.

GHP can be thought of as a Brayton FoM because a perfect Brayton cycle will have no loss of GHP, whereas any departure from the ideal Brayton cycle will appear as a loss in GHP. It appears to be most useful for analysis of gas-turbine power generation units and turboshaft engines and is measured relative to a prescribed ambient pressure but not ambient temperature. However, GHP counts exhaust residual KE as a loss even though this portion of the exhaust GHP is inherently unavailable to jet propulsion applications if the cycle is taken as given. Finally, GHP was shown to be a special case of exergy wherein only mechanical equilibrium with the environment is enforced.

Thrust work potential produces results suggesting that it is a pure jet propulsion FoM because it is a direct index on the ability to produce thrust work. In effect, thrust work potential is a measure of ability to project thrust work into the Earth-fixed reference frame and is related to GHP through propulsive efficiency. Thus, thrust work potential is a special case of GHP, and by extension, a special case of exergy.

Finally, the results given here suggest that the proper choice of thermodynamic FoM is dependent on the type of machine being analyzed and the intentions of the analyst. If the objective is to understand the absolute loss relative to the maximum work allowed by the second law of thermodynamics, then exergy is an appropriate tool to use. If the objective is component optimization for minimum loss of usable work potential, then the focus should be on minimization of loss in GHP for a simple gas turbine, or thrust work potential for propulsors. As a general rule, inherently unavailable work potential should not be counted as a loss chargeable to the machine when optimizing to deliver a specific type of work output. Therefore, the proper choice of loss FoM inherently depends on how one defines useful work.

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